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PAPER

Relativity without a Time Dimension: Absolute Time, Co-expansion along the Flow of Space, and a Positive-Definite Geometry of Gravitation

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Abstract

Special relativity is normally written with time as a fourth dimension. We show that its kinematics can instead be written with a fourth spatial axis y and no time dimension. Motion along the four axes is the fundamental rate of change, and the inclination of a body's path into y carries the kinematics: its slope is $\beta\gamma$, its sine is the speed, and its cosine is the rate of a clock carried along it. Time is a ratio of distances, light enters only as a causal sphere, every inertial observer recovers one absolute time, and every interval is a sum of squares, so the geometry has signature $(+,+,+,+)$. Gravity uses the same inclination. Space flows outward from every mass at the escape velocity, and matter at rest relative to the mass co-expands as a body moving through the flowing space does: lengths along the flow scale with the Lorentz factor of the flow, clocks with its inverse, and lengths across the flow are unchanged. This co-expansion is the same for all matter, which is the origin of the equivalence principle. On flat slices of absolute time the rule that sets the flow is the Hamiltonian constraint of the Einstein equations, and the inclination of space into y is Flamm's paraboloid. The geometry is that of general relativity, which therefore gains an absolute time and a positive-definite form.

1 Introduction

In special relativity time is a fourth dimension of spacetime, and its rate is tied to the speed of light. Time does two jobs there: it locates events, as a fourth coordinate does, and it orders change, as a rate does. This paper keeps both jobs and removes time as a dimension. The fourth coordinate becomes a spatial one, a density axis y that extends ordinary space $\mathbb{R}^3$ in the manner of an unwrapped, non-compact Kaluza–Klein dimension [34, 35]. The rate of change becomes motion, which is primitive and is not measured against anything else. A body moves along four axes: the three of $\mathbb{R}^3$ and the density axis, along which it is carried by its own motion.

One geometric object carries the argument: the inclination of a path through $\mathbb{R}^3 \times y$. Section 2 shows that it contains the kinematics of special relativity (SR) [20]. Its slope is $\partial_x y = \beta\gamma$, so that $\gamma = \sqrt{1+(\partial_x y)^2}$; its sine is the speed; and its cosine is the coordinate density, which is also the rate of a clock carried along the path. Time readings are ratios of distances along the path, and the speed of light enters only as the speed of the causal sphere that spreads from each event. Every inertial observer can then recover one absolute time, and the relativity of simultaneity becomes a displacement along y.

Section 3 builds gravity from the same inclination. Space flows outward from every mass, as in the Hubble flow, at the escape velocity βc, with $\beta^2 = 2GM/(c^2R)$ and R the measured radius. Matter held at rest relative to the mass moves through the flowing space, and it responds as a moving body does in SR: its lengths along the flow are scaled by the Lorentz factor γ of the flow and its clocks by $1/\gamma$, while lengths across the flow are unchanged. We call this co-expansion along the flow. It is the same for all matter, so free fall cannot depend on composition, and the equivalence principle follows from the geometry. On the flat slices of absolute time the flow rule is the Hamiltonian constraint of the Einstein equations, the space around a mass is inclined into y as Flamm's paraboloid, its height is the offset between absolute and static clocks, and the geometry of

an isolated mass is exactly the Schwarzschild exterior. Light and matter then move without a time coordinate (section 4). Section 5 places the density axis in a five-dimensional Kaluza–Klein metric, section 6 compares the construction with experiment, and section 7 states what has been shown.

The construction joins several established lines of work. Mansouri and Sexl showed that absolute simultaneity is compatible with the predictions of SR [41], and the neo-Lorentzian tradition treats relativistic effects as properties of motion relative to a preferred frame [8, 14]. Euclidean relativity [46, 2] also uses a positive-definite four-dimensional geometry. Induced-matter theory uses a non-compact fifth dimension [60, 48], and the Campbell–Magaard theorem allows any analytic solution of the Einstein equations to be embedded locally in five dimensions [15, 39]. The outward flow is the time reverse of the river model of black holes [30], and the view that time is abstracted from change is that of ephemeris time [7]. The inclination of space into y around a mass is Flamm's embedding of the Schwarzschild space [24]. What this paper adds is the identification of the Lorentz factor, the speed, the clock rate and the synchronization offset with one inclination in $\mathbb{R}^3 \times y$, in SR and in gravity alike, a positive-definite form of both, and co-expansion along the flow as the geometric origin of the equivalence principle.

The construction is a reformulation, not a modification. Its kinematics is that of SR, and its field equations are those of general relativity written on the slices of absolute time, so it makes the predictions of general relativity wherever that theory has been tested. What it changes is the account of time: time is not a dimension, the rate of change is motion, every interval is a sum of squares, and simultaneity is absolute. An earlier version, in which matter co-expanded in every direction, departed from general relativity at second order; its neutron stars violate the strong equivalence principle by far more than the triple pulsar PSR J0337+1715 allows (subsection 6.2), and co-expansion along the flow is the choice that both experiment and SR select.

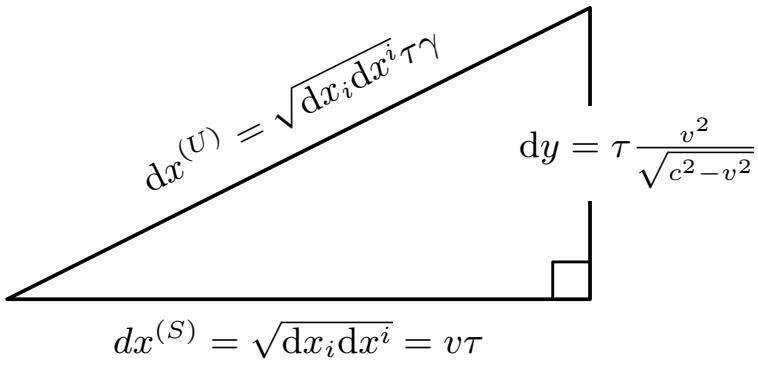


Figure 1. A path element through $\mathbb{R}^3 \times y$: the advance dx through $\mathbb{R}^3$, the advance dy along the density axis, and the path length $ds = dx^{(U)} = \gamma\, dx$. The inclination ψ of the path satisfies $\tan\psi = \partial_x y = \beta\gamma$ (subsection 2.2).

1.1 The Lighthouse & The Clock-Tower

The picture can be stated as a thought experiment. Let **S** be the absolute frame, and let two observers, A and B, agree that B will travel at an agreed speed v relative to **S** between two points p_1 and p_2 at rest in **S**, a distance d apart. A remains at p_1. At p_2 stands a clock tower large enough to be read from p_1. Its face is read by the light it sends out: a lighthouse, whose light spreads from each event as a sphere of radius $c\,\Delta t$.

We make two assertions.

1. To keep a classical simultaneity in $\mathbb{R}^3$, A and B must agree on the time at which B reaches p_2.
2. To agree with the measured dilation of moving clocks [53, 25, 6, 5, 11], the clock carried by B must advance by $d/(\gamma v)$ on the trip.

Both hold if the motion of B is a path through $\mathbb{R}^3 \times y$ (subsection 2.2). Along it B advances d through $\mathbb{R}^3$ and $\beta\gamma d$ along y, so the path has length γd. The absolute time of the trip,

$$\Delta\tau = \frac{d}{v} = \frac{\gamma d}{\gamma v}, \tag{1}$$

is the same whether it is formed from the absolute distance and speed, as A does, or from the path length and the rate γv along the path, as B does. In the second form the speed is divided by the coordinate density $1/\gamma$: a speed referred to the path dilates inversely with the coordinate density. The clock of B reads $d/(\gamma v)$ (subsection 2.2), which satisfies the second assertion. The light from the clock tower does not define the time of the trip; it is how the time is read. Because every reading is carried by light, time appears to pass at c. A photon is the limit of the construction in which the path turns entirely along y and the coordinate density vanishes (subsection 2.2).

2 Motion without Time

This section builds the kinematics of SR from the two properties of time that the model keeps: a fourth displacement coordinate, the position along y, and a driving rate of change, motion. Throughout, **S** is the absolute frame in which $\mathbb{R}^3$ is at rest, and τ is its time. τ is not an independent coordinate: subsection 2.2 obtains $c\tau$ as a ratio of lengths along a path, so seconds enter only through the value of c. We also require that, in a frame co-moving along y, mechanical equations reduce to their Newtonian forms, with only the relativistic part of each magnitude extending along y. Table 1 lists the quantities used and their standard counterparts.

Table 1. Quantities of this paper and their standard counterparts

This paper	Standard counterpart
ψ, $\tan\psi = \partial_x y$	inclination of a path, or of space around a mass, in $\mathbb{R}^3 \times y$; $\psi = \mathrm{gd}(\eta)$, the Gudermannian of the rapidity
$\sin\psi = \beta = v/c$	speed; in gravity the flow velocity, which is the shift vector ("river velocity") and the escape velocity from the enclosed mass
$\cos\psi = 1/\gamma$	coordinate density; the rate of a moving clock, and of a static clock outside a mass (co-expansion along the flow)
$c\,d\tau = dx\,ds/dy$, $c\,dt_U = dx^2/dy$	absolute time and proper time of a moving clock
$y(R)$, $\partial_R y = \beta\gamma$	inclination of space around a mass along the flow (Flamm's paraboloid)
R	measured (areal) radius; on the flat absolute slices also the absolute radius
$\partial_\tau y = v^2/\sqrt{c^2 - v^2}$	dy/dT along a worldline at rest in the outward flow

2.1 The Fourth Coordinate and the Lorentz Factor

Let a body move at speed v relative to **S**, and let the additional coordinate grow along its motion as

$$y = \tau \frac{v^2}{\sqrt{c^2 - v^2}}. \tag{2}$$

With $x = v\tau$ this gives

$$\partial_\tau y = \frac{v^2}{\sqrt{c^2 - v^2}} = \frac{\gamma v^2}{c}, \tag{3}$$

$$\partial_x y = \frac{v}{\sqrt{c^2 - v^2}} = \beta\gamma, \tag{4}$$

and $v\,\partial_x y = \partial_\tau y$. Squaring equation (4) and using $\gamma^2 = 1/(1-\beta^2)$ gives

$$\gamma = \sqrt{1 + (\partial_x y)^2}, \tag{5}$$

so the Lorentz factor is the slope of the additional coordinate. The length of a path element through $\mathbb{R}^3 \times y$ is then

$$ds^2 = dx^2 + dy^2 = \gamma^2 dx^2. \tag{6}$$

This is the separation $dx' = \gamma\,dx$ that the Lorentz transformation assigns, in the frame **U** moving at v, to two events separated by dx at equal τ. The same transformation gives $c\,dt' = -\gamma(v/c)\,dx = -dy$ for these events, and the interval is preserved as a sum of squares, $dx'^2 = dx^2 + c^2 dt'^2 = dx^2 + dy^2$. Subsection 2.3 extends this from equal-τ separations to arbitrary events. The relation to the rapidity η is elementary: $\gamma = \cosh\eta$ and $\partial_x y = \sinh\eta$, so equation (5) is the identity $\cosh^2\eta = 1 + \sinh^2\eta$.

2.2 Motion as Paths through $\mathbb{R}^3 \times y$

A motion is a path through $\mathbb{R}^3 \times y$. It carries no time parameter; its only property at each point is its direction. Let ψ be the inclination of the path above $\mathbb{R}^3$ (figure 1). Equation (4) gives $\tan\psi = \partial_x y = \beta\gamma$, and hence

$$\sin\psi = \frac{dy}{ds} = \beta, \qquad \cos\psi = \frac{dx}{ds} = \frac{1}{\gamma}. \tag{7}$$

The two direction cosines are the rates of the model. The speed is the advance along y per unit path, $\sin\psi$: a body is propelled along the density axis by its motion, and its speed is that propulsion. The fraction $\cos\psi$ of the path that lies in $\mathbb{R}^3$ is the coordinate density, the number of absolute coordinates per measured length, and it is also the rate of a clock carried along the path.

Time readings as ratios of distances. Along a path element with advances dx and dy, the absolute time and the reading of a clock carried along the path are

$$c\,d\tau = \frac{dx\,ds}{dy} = \frac{dx}{\sin\psi}, \qquad c\,dt_U = \frac{dx^2}{dy} = \frac{dx}{\tan\psi}. \tag{8}$$

They give $dx = v\,d\tau$, the moving-clock rate $dt_U = d\tau/\gamma$, and $c\,dt_U = \gamma c\,d\tau - dy$ along the path, which are items 4 and 1 of the Proposition in subsection 2.3. Neither reading is an independent coordinate. Both are lengths formed from the path.

The causal sphere. Light spreads from each event as a sphere of radius $c\,d\tau$. While a body advances dx through $\mathbb{R}^3$ the sphere reaches $c\,d\tau = dx/\sin\psi$, so

$$\frac{dx}{c\,d\tau} = \sin\psi \le 1, \tag{9}$$

and every body remains inside the causal sphere of its own past events. This is the only place where c enters. The path itself does not advance at c: its length per unit of light distance, $ds/(c\,d\tau) = \beta\gamma = \tan\psi$, takes any value. Time is read off the causal sphere, so it appears to pass at c; the rate of change that the model identifies with time is the motion along the four axes, which is not bound to that rate. Whether the two rates can be told apart by measurement is discussed in subsection 7.2.

Composition. With $\tanh\eta = \beta$, the inclination is the Gudermannian of the rapidity, $\psi = \mathrm{gd}(\eta) = \arctan(\sinh\eta)$. Collinear motions compose by adding rapidities, so $\sin\psi_W = \tanh(\eta_1 + \eta_2) = (\beta_1 + \beta_2)/(1 + \beta_1\beta_2)$, the relativistic composition of velocities.

Limits. A body at rest has no path ($dx = dy = 0$), and equation (8) gives it no time reading of its own. Its time must come from other change: its internal motion, or the motion of everything else, as in ephemeris time [7]. Light is the limit $\psi \to 90°$, in which the coordinate density $\cos\psi$ vanishes, the clock reading $c\,dt_U$ goes to zero, and $c\,d\tau$ goes to dx, the radius of the causal sphere. The limit is consistent but degenerate.

Relation to Euclidean relativity. Euclidean relativity [46, 2] writes $(c\,d\tau)^2 = dx^2 + (c\,d\tau_p)^2$, with the proper time τ_p as a fourth spatial coordinate, and postulates that everything moves through that space at c. Its worldline makes the angle ψ with the proper-time axis, $\tan\psi = dx/(c\,d\tau_p) = \beta\gamma$. The path triangle $(dx, dy; ds)$ of this paper is therefore similar to its triangle $(c\,d\tau_p, dx; c\,d\tau)$, scaled by $\tan\psi$. The postulate is not used here, and the constructions differ: the path of this paper advances at $ds/(c\,d\tau) = \tan\psi$, not at c.

The next subsection uses these readings to show that every inertial observer recovers the same absolute time.

2.3 Classical Simultaneity in $\mathbb{R}^3$

Subsection 2.1 treated two events at equal τ. Here, once the displacement along y is included, every inertial observer assigns the same absolute time τ to every event, and the relativity of simultaneity is entirely a displacement along y.

Let events be labelled by an absolute position x in $\mathbb{R}^3$ (one axis is shown) and by the absolute time τ, both referred to the frame **S**. An observer **U** moving at velocity v relative to **S** assigns to each event the displacement

$$y_U(x) = \gamma \frac{v}{c} x, \tag{10}$$

which is equation (4) integrated at fixed velocity.

Proposition. For every event (x, τ) and every inertial observer **U** moving at v relative to **S**:

1. the Lorentz coordinates of **U** are
$$x_U = \gamma\,(x - v\tau)\,, \qquad c\,t_U = \gamma c\,\tau - y_U(x); \tag{11}$$
2. every observer recovers the same absolute time,
$$\tau = \frac{c\,t_U + y_U(x)}{\gamma c}; \tag{12}$$
3. two events with $\Delta\tau = 0$ and any separation Δx satisfy $c\,\Delta t_U = -\Delta y_U$;
4. along the worldline of **U**, $x = v\tau$, the clock of **U** advances by $dt_U = d\tau/\gamma$;
5. for an observer **W** moving at v_2 relative to **U**, items 1 to 4 hold with v replaced by the velocity of **W** relative to **S**, $v_W = (v + v_2)/(1 + vv_2/c^2)$.

Proof. 1. With τ as the time coordinate of **S**, the Lorentz transformation gives $x_U = \gamma(x - v\tau)$ and
$$c\,t_U = \gamma\left(c\tau - \frac{v}{c}x\right) = \gamma c\,\tau - \gamma\frac{v}{c}x = \gamma c\,\tau - y_U(x), \tag{13}$$
using equation (10).

2. Solving equation (11) for τ gives equation (12). The right-hand side uses only quantities assigned by **U** and its velocity relative to **S**, and it returns the same τ for every v.

3. Subtracting equation (11) for two events at equal τ gives $c\,\Delta t_U = -\gamma(v/c)\Delta x = -\Delta y_U$.

4. On $x = v\tau$, $y_U = \gamma v^2\tau/c$, so
$$c\,t_U = \gamma c\,\tau\left(1 - \frac{v^2}{c^2}\right) = \frac{c\,\tau}{\gamma}. \tag{14}$$
The clock of **U** loses, relative to $\gamma c\,d\tau$, exactly its own advance $dy_U = \gamma(v/c)\,v\,d\tau$ along y.

5. Collinear boosts compose by addition of rapidities, so the boost at v followed by the boost at v_2 is the boost at v_W with $\gamma_W = \gamma\gamma_2(1 + vv_2/c^2)$. Items 1 to 4 then follow with v_W and $y_W(x) = \gamma_W(v_W/c)\,x$. □

The conventional time difference $\Delta t_U = -\gamma v\Delta x/c^2$ between two events at the same τ is therefore not a difference in absolute time. It is the displacement Δy_U that **U** assigns to their separation, and all of $\mathbb{R}^3$ at a given τ is one instant for every observer. Item 4 obtains the slowing of a moving clock from its displacement along y; it does not insert time dilation separately. In the language of subsection 2.2, the clock rate $1/\gamma$ is the coordinate density $\cos\psi$ of the path. For $\Delta\tau = 0$ the absolute separation is the same for every observer, through the embedding relation $\Delta x_U^2 = \Delta x^2 + \Delta y_U^2$, a sum of squares with the absolute length as the invariant leg. The separation Δx_U measured by **U** is the hypotenuse of the graph of y_U over $\mathbb{R}^3$ and depends on the observer, while its projection Δx onto $\mathbb{R}^3$ does not. With $w = c\tau$, a length, item 1 reads $x_U = \gamma\,(x - \beta w)$ and $w_U = c\,t_U = \gamma w - y_U$. The transformation acts on lengths alone, and its invariant, $x_U^2 + w^2 = x^2 + w_U^2$, a sum of squares on each side, is the Minkowski interval written in them; the geometry of subsection 2.2 reads the same transformation as the inclination of a path.

The frame **S** is an assumption of the model. Nothing in the kinematics singles it out, and every local experiment gives the same result in it as in any other inertial frame. We identify it with the frame of the flow of space: the cosmic rest frame on large scales, whose velocity relative to an observer is measured by the dipole of the cosmic microwave background, and the outward flow of section 3 near a mass. The next section shows that the same inclination, and the same offset along y, appear in gravity once matter co-expands along the flow of space.

3 Co-expansion and Gravity

In SR the inclination of a path is set by the speed of the body. In gravity it is set by the flow of space, and the geometry needs one further statement, about how matter held at rest relative to a mass responds to the flow. This section states it, shows that it gives the equivalence principle, and derives the geometry of space around a mass, which turns out to be that of general relativity.

3.1 Co-expansion along the Flow

In SR an observer moving relative to **S** assigns the length $ds = \gamma\, dx$ to an absolute separation dx along its motion (equation (6)), assigns the absolute length to a separation across its motion, and its clock runs at $1/\gamma$ (subsection 2.3). Matter held at rest relative to a mass is in the same situation, because space flows past it at the speed βc (subsection 3.2).

Postulate (co-expansion along the flow). All matter at rest relative to a mass co-expands with the change in y as a body moving through the flowing space does. Every material length along the flow, of a ruler, an atom or the Earth, is scaled by the Lorentz factor γ of the flow; every clock runs at the coordinate density $1/\gamma$; and lengths across the flow are unchanged.

Let R be the measured radius around the mass, so that $2\pi R$ is the measured circumference of a sphere. Measured with co-expanding rulers and clocks, a clock that moves through a measured displacement while the static time advances by dt reads $d\tau$, with

$$\frac{c^2 dt^2}{\gamma^2} = c^2 d\tau^2 + \gamma^2 dR^2 + R^2 d\Omega^2. \tag{15}$$

Every term is a square. The static time, read through the coordinate density $1/\gamma$, is the hypotenuse, and the clock reading and the measured displacement are the legs, as in the path triangle of subsection 2.2, where equation (8) gives $(c\, d\tau)^2 = (c\, dt_U)^2 + dx^2$. Moving $c^2 d\tau^2$ to the other side gives the Lorentzian form of the same relation. We keep the positive-definite form throughout: the geometry has signature $(+,+,+,+)$, and no quantity in it is a time dimension.

The scaling is the same for every kind of matter. The motion of a body in equation (15) therefore cannot depend on what the body is made of, and the universality of free fall is a property of the geometry rather than a separate principle. Co-expansion is in this sense the origin of the equivalence principle. A body released at rest relative to the mass is no longer held against the flow, and no longer co-moves along y with the matter around it; relative to that matter, which stays co-expanded, it falls. Co-expansion is not observable locally, because every ruler and every clock at a point co-expands by the same factor. Its exponents, -1 for clocks, $+1$ along the flow and 0 across it, are those of a moving body in SR (subsection 6.3).

3.2 The Outward Flow

Let T be the absolute time and $\mathbf{x}$ the position in absolute space, with $R = |\mathbf{x}|$ measured from a mass M. Space flows radially outward, in the same direction as the Hubble flow, at the velocity $\mathbf{W} = \beta c\, \hat{\mathbf{R}}$ relative to an observer held at fixed R. In the absolute frame a clock that moves through the absolute displacement $d\mathbf{x}$ while the absolute time advances by dT reads $d\tau$, with

$$c^2 dT^2 = c^2 d\tau^2 + |d\mathbf{x} - \mathbf{W}\, dT|^2, \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}, \tag{16}$$

where $|\cdot|$ is the Euclidean length of flat absolute space. This is the clock reading of SR (subsection 2.2) with the flowing space in place of **S**: the absolute time is the hypotenuse, and the clock reading and the displacement relative to the flowing space are the legs. The absolute slices are flat, so the absolute radius and the measured radius are the same, and an observer carried by the flow, $d\mathbf{x} = \mathbf{W}\, dT$, reads $d\tau = dT$ and moves freely. Written in Lorentzian form, equation (16) is the Painlevé–Gullstrand form of the Schwarzschild metric [49, 29, 42], with its flow reversed to point outward. The inward form is the river model [30]. The outward direction is a postulate of the model.

The flow is set by the measured radius. The flow speed is the escape velocity from the mass,

$$\beta^2 = \frac{2GM}{c^2 R}, \qquad \gamma = \frac{1}{\sqrt{1 - 2GM/(c^2 R)}}, \tag{17}$$

and subsection 3.3 shows that this rule is a field equation. At the surface of the Earth space streams outward at the escape velocity, about 11.2 km s^{-1}. Because lengths along the flow co-expand, the proper radius of a body exceeds its measured radius: for a body of uniform density by $GM/(3c^2)$ to first order, which for the Earth is about 1.5 mm. Neither is visible locally, because every ruler on the Earth co-expands with it. At $R = 2GM/c^2$ the flow reaches the speed of light.

The Hubble flow. For flat FLRW cosmology, $R = a(t)r$ gives the same outward form with $\beta_H = HR/c$, and the Hubble radius c/H, where $\beta_H = 1$, is its apparent horizon [22, 23]. With $m = \frac{4\pi}{3}\rho R^3$ and the Friedmann equation $H^2 = 8\pi G\rho/3$, the Hubble flow obeys $\beta_H^2 = 2Gm/(c^2R)$, the relation of equation (17) with the enclosed mass, and its Newtonian counterpart is the zero-energy condition of Milne and McCrea [44, 43]. Binding and expansion are one outward flow with different distributions of mass, and subsection 3.3 shows that both relations are the same field equation. The co-expansion form of an expanding universe is not developed further in this paper.

3.3 The Field Equations

The flow and the measured geometry obey the Einstein equations, written on the slices of absolute time. In this $3+1$ form [4] the geometry is described on each slice by the rate N of absolute time, the flow $\mathbf{W}$ and the metric h_{ij} of the slice, and the motion of the slice through the geometry by the rate of strain

$$K_{ij} = \frac{1}{2N}\left(\partial_T h_{ij} + D_i W_j + D_j W_i\right). \tag{18}$$

The equations are four constraints on each slice and evolution equations from one slice to the next. For the outward flow of equation (16), $N = 1$, $h_{ij} = \delta_{ij}$ and K_{ij} is the rate of strain of the flow, with $K^R{}_R = c\,d\beta/dR$ and $K^\theta{}_\theta = K^\phi{}_\phi = \beta c/R$. The Hamiltonian constraint, $K^2 - K_{ij}K^{ij} = 16\pi G\rho/c^2$ on a flat slice, then reads

$$\frac{d}{dR}\left(R\beta^2\right) = \frac{8\pi G}{c^2}\,\rho R^2, \qquad \beta^2 = \frac{2Gm(R)}{c^2 R}, \tag{19}$$

with $m(R)$ the mass within R and ρ the density of energy divided by c^2. Outside matter it gives equation (17), and for the Hubble flow it is the Friedmann equation. The rule that sets the flow by the measured radius is therefore the Hamiltonian constraint of the Einstein equations on flat absolute slices. The radial momentum constraint holds identically for any radial flow. For pressureless matter the evolution equations give the marginally bound solutions of Lemaître and Tolman [36, 57]. Inside a body supported by pressure the rate of absolute time is no longer one, and hydrostatic equilibrium is that of Tolman, Oppenheimer and Volkoff [58, 47].

Status. The field equations are those of general relativity. Every prediction of general relativity therefore holds in this construction, for isolated masses, for self-gravitating bodies, for moving and rotating sources and for gravitational radiation, and section 6 compares them with experiment. What the construction adds is the reading of the geometry: absolute time, the inclination of space into y, and co-expansion along the flow as the origin of the equivalence principle.

3.4 The Inclined Space

An observer held at fixed R moves inward relative to space at $-\beta c$. As for a moving observer in SR, it measures the radial separation $d\ell = \gamma\, dR$ along its motion through the flow and the angular separation $R\,d\Omega$ across it: this is the co-expansion of subsection 3.1, and equation (22) below derives it from the flow.

The space around a mass is inclined into y exactly as a path is (figure 1), with the flow speed in place of the speed of a body:

$$\partial_R y = \beta\gamma, \qquad \sin\psi = \beta, \qquad \cos\psi = \frac{1}{\gamma} = \frac{dR}{d\ell}. \tag{20}$$

The radial measured length is the arc length of the graph of y over the measured radius, $d\ell^2 = dR^2 + dy^2 = \gamma^2 dR^2$, and the height of the graph is

$$y(R) = 2\sqrt{\frac{2GM}{c^2}\left(R - \frac{2GM}{c^2}\right)}, \tag{21}$$

Flamm's paraboloid [24]. Its induced metric, $\gamma^2 dR^2 + R^2 d\Omega^2$, is the measured geometry of equation (15): the space of a static observer is a surface in the Euclidean space $\mathbb{R}^3 \times y$, inclined along the flow and level across it. Co-expansion along the flow is this inclination, and it needs no factor beyond it.

The height is the synchronization offset. The change of time $c\,dT = c\,dt - \beta\gamma^2\,dR$ takes the absolute form (16) to the static form

$$\frac{c^2dt^2}{\gamma^2} = c^2d\tau^2 + \gamma^2dR^2 + R^2d\Omega^2, \tag{22}$$

which is equation (15). The offset between the absolute time T and the static time t is

$$c\,(dT - dt) = -\beta\gamma^2\,dR = -\gamma\,(\beta\gamma\,dR) = -\gamma\,dy. \tag{23}$$

In the proper time of the static observer, $c\,(dT - dt)/\gamma = -dy$: the offset is the height itself. The static observer moves inward relative to space at $-\beta c$, so light it sends outward travels with the flow at $c(1+\beta)$ and returns against it at $c(1-\beta)$. Its synchronization parameter is $\epsilon = (1-\beta)/2$ [52], the value $\frac{1}{2}(1+u/c)$ of an observer moving at u, and $dy = (1-2\epsilon)\,d\ell$. The structure is that of item 3 of the Proposition: absolute space supplies the observer-independent base dR, the static proper length $\gamma\,dR$ is the arc length of the graph, and a separation along y is a difference in clock setting at one absolute time.

Coordinate density. Along the flow equation (22) gives

$$\frac{dR}{d\ell} = \frac{1}{\gamma} = \cos\psi = \frac{d\tau_{\text{static}}}{dt}, \tag{24}$$

and across the flow the coordinate density is one. Along the flow the number of absolute coordinates per measured length equals the rate of a static clock: in vacuum the coordinate density and the rate of time are one quantity, as for a moving clock in SR (subsection 2.2).

3.5 The Exact Metric

With equation (17), the static form (22) is, in Lorentzian form,

$$-c^2d\tau^2 = -\left(1 - \frac{2GM}{c^2R}\right)c^2dt^2 + \frac{dR^2}{1 - 2GM/(c^2R)} + R^2d\Omega^2, \tag{25}$$

the Schwarzschild exterior [45], which solves the vacuum Einstein equations. Every observable of an isolated mass therefore has its general-relativistic value at all orders: the perihelion advance, light deflection, the Shapiro delay, the gravitational redshift, the photon sphere at $R = 3GM/c^2$, the innermost stable circular orbit at $R = 6GM/c^2$ and the size of a black hole's shadow.

The surface $R = 2GM/c^2$. There the flow reaches the speed of light and the static clock rate vanishes. In the outward flow, the region inside it is the time reverse of a black-hole interior, through which space streams outward faster than light. Gravitational collapse, and which flow describes a collapsed body, are not treated in this paper.

3.6 Several Masses

The Einstein equations are nonlinear, so the flows of several masses do not add. In the weak field the condition $\gamma^2 = 1 + |\nabla h|^2 = 1 - 2\Phi/c^2$ is the eikonal equation $|\nabla h|^2 = -2\Phi/c^2$ for the height of the inclined space, and where the outward flows of two bodies meet head-on, ∇h changes direction across a surface that carries no vorticity and exerts no force. Beyond that order the measured geometry of several masses is not the metric induced on a single hypersurface $y = h(\mathbf{x})$: the Gauss equation can be solved point by point for the second derivatives of h, but the result fails the Codazzi equations, so it is not the Hessian of any height (the accompanying code checks this numerically). The inclination into y is then a local description, and the Einstein equations determine the geometry. For masses momentarily at rest the initial slice is conformally flat, with a conformal factor that superposes linearly [13]. How the absolute slices of different bodies join, for example whether the surfaces of two bodies lie in one slice, is not fixed by the construction. General relativity admits any such slicing, and no prediction depends on the choice.

3.7 The Rate of Change of Coordinate Density

The two properties of time that the model keeps, a fourth displacement coordinate and a rate of change, have a counterpart in the flow: the coordinate density of equation (24), and the rate at which the flow changes it. The flow changes the volume of a region of absolute space carried with it

at the fractional rate $\theta = \nabla \cdot \mathbf{W}$, the trace K of the rate of strain (18). On the flat absolute slices around a mass,

$$\theta = \frac{1}{R^2}\frac{d}{dR}\left(R^2\beta c\right) = \frac{3\beta c}{2R}, \tag{26}$$

made of a radial compression at the rate $\beta c/(2R)$ and a transverse stretching at $\beta c/R$ in each direction, which balance in the Hamiltonian constraint, $K^2 = K_{ij}K^{ij}$ in vacuum. For the Hubble flow, $\mathbf{W} = H\mathbf{R}$, the flow is pure expansion, $\theta = 3H$, and the constraint and Raychaudhuri equations [51] written for the flow are the Friedmann and acceleration equations.

4 Paths without Time

Subsection 2.2 removed time from SR. For a steady flow the same can be done in gravity: the time coordinate can be eliminated exactly, leaving the paths of light and of bodies through space, with time readings derived from the paths afterwards. Here "steady" means that the flow does not change along itself, so that the geometry is stationary. It is not an assumption that time is periodic or compact: no coordinate is wrapped.

Light: the causal sphere carried by the flow. In the absolute frame, equation (16), light has $d\tau = 0$ and obeys $|d\mathbf{x} - \mathbf{W}\,dT| = c\,dT$ in flat absolute space. The causal sphere spreads at c relative to the outward-flowing space while being carried by it. This is Zermelo's navigation problem on Euclidean $\mathbb{R}^3$, and its travel time is the length of a Randers–Finsler metric [27]. For $W < c$,

$$dT = \frac{-\mathbf{W}\cdot d\mathbf{x} + \sqrt{\left(\mathbf{W}\cdot d\mathbf{x}\right)^2 + \left(c^2 - W^2\right)|d\mathbf{x}|^2}}{c^2 - W^2}. \tag{27}$$

Radially it gives $dR/dT = c(1+\beta)$ outward and $c(1-\beta)$ inward, as in subsection 3.4. In the static form, equation (22), light has $c\,dt = \gamma\,d\ell$ with $d\ell$ the measured length in the inclined space, and obeys Fermat's principle with the optical metric $\gamma^2\left(\gamma^2 dR^2 + R^2 d\Omega^2\right)$:

$$c\,dt = n\,d\ell, \qquad n = \gamma. \tag{28}$$

Light therefore moves through the inclined space as through a medium of refractive index γ, the slowing of static clocks. The two travel times differ by an exact differential,

$$c\,dT = c\,dt - \beta\gamma^2\,dR = c\,dt - \gamma\,dy, \tag{29}$$

which is the synchronization offset of equation (23). An exact differential does not change the paths that extremize the travel time, so the two descriptions give the same light paths, and the height of the inclined space is the bridge between them.

Bodies with $v \neq c$. A body does not move with the causal sphere, and its paths are not those of light. In a static field its paths at fixed energy E per unit rest energy are the geodesics of the Jacobi metric [26], which for equation (22) is

$$J = \left(E^2\gamma^2 - 1\right)\left(\gamma^2 dR^2 + R^2 d\Omega^2\right). \tag{30}$$

It is a positive-definite metric on the inclined space with no time coordinate, and time is recovered afterwards along the path. In the absolute frame the corresponding metric is of Randers type [16], and the reduced momenta in the two frames differ by E times the same exact differential, so bodies also follow the same paths in both.

Orbits. With $u = 1/R$, angular momentum L per unit mass and $G = c = 1$, the equatorial geodesics of equation (30) give (appendix A)

$$\left(\frac{du}{d\phi}\right)^2 = \frac{E^2 - (1-2Mu)\left(1 + L^2u^2\right)}{L^2}, \tag{31}$$

the Schwarzschild orbit equation. The perihelion advance is $\Delta\phi = 6\pi GM/[a(1-e^2)c^2]$ to leading order, circular orbits have $L^2 = MR^2/(R-3M)$, the innermost stable one is at $R = 6M$, and light circles at $R = 3M$ with critical impact parameter $b_c = 3\sqrt{3}M$. For light,

$$\left(\frac{du}{d\phi}\right)^2 = \frac{1}{b^2} - u^2 + 2Mu^3, \tag{32}$$

and the deflection of a ray with impact parameter b is

$$\alpha = \frac{4M}{b} + \frac{15\pi}{4}\frac{M^2}{b^2} + O\left(\frac{M^3}{b^3}\right). \tag{33}$$

Scope. The elimination is exact for the steady flow around an isolated mass. An expanding universe with varying H is navigation in a wind that changes along the flow, and its time is not reduced to a fixed metric here. What the elimination shows is that, around a mass, nothing in the motion of light or of bodies requires a time dimension: the paths lie in flat absolute space carried by the flow, or equivalently in the inclined space with an index of refraction γ; the causal sphere supplies c, and time is read from the paths.

The density axis has so far been a flat fourth axis. Section 5 asks how it fits a five-dimensional metric.

5 The Unwrapped Kaluza–Klein Metric

Sections 2 to 4 used y as a fourth spatial axis with a flat metric. This section places it in a five-dimensional metric. The off-diagonal components then contain the potential generated by y and the gravitomagnetic field of rotating sources, and the diagonal component gives a scalar sector.

5.1 The Five-Dimensional Metric

We represent the additional coordinate through the metric component G_{44} of a five-dimensional metric,

$$dS^2 = g_{\mu\nu}dx^\mu dx^\nu + G_{44}\,dy^2, \qquad G_{AB} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & G_{44} \end{pmatrix}. \tag{34}$$

The coordinate y is not compactified. For a path, $dx^2 + G_{44}\,dy^2$ with $dy = \beta\gamma\,dx$ reproduces $ds^2 = \gamma^2 dx^2$ only for $G_{44} = 1$. In gravity the measured geometry is the induced metric of Flamm's paraboloid, $d\ell^2 = dR^2 + G_{44}\,dy^2 + R^2 d\Omega^2$ with $\partial_R y = \beta\gamma$ (equation (20)), which requires the same normalization, $G_{44} = 1$, to all orders: co-expansion along the flow is the inclination of space into y, with no further factor.

Four-dimensional gravity from a non-compact dimension. A flat, non-compact extra dimension does not by itself give four-dimensional gravity: if the field equations were obtained from a five-dimensional action integrated over y, the effective Newton constant would diverge. Induced-matter theory [60, 48] works instead with the five-dimensional field equations under the cylinder condition $\partial_y g_{\mu\nu} = \partial_y G_{44} = 0$, and brane localization [50] is another route. A five-dimensional action for the model remains open.

5.2 The Four-Potential and Rotation

In Kaluza–Klein geometry the off-diagonal components of the metric are the four-potential. Restoring them,

$$dS^2 = g_{\mu\nu}\,dx^\mu dx^\nu + G_{44}\,(dy + \mathcal{A}_\mu dx^\mu)^2\,. \tag{35}$$

The line element is invariant under $y \to y + \Lambda(x)$, $\mathcal{A}_\mu \to \mathcal{A}_\mu - \partial_\mu\Lambda$. The density axis supplies a natural potential, evaluated in the rest frame of the source,

$$\mathcal{A}_\mu \equiv \partial_\mu y, \qquad \mathcal{A}_0 = 0, \qquad \mathcal{A}_i = \beta\gamma\,\hat{n}_i, \tag{36}$$

and three identities follow:

$$\gamma^2 = 1 + \mathcal{A}_i\mathcal{A}^i, \tag{37}$$

$$\eta^{\mu\nu}\mathcal{A}_\mu\mathcal{A}_\nu = \beta^2\gamma^2 = \gamma^2 - 1, \qquad \mathcal{A}_\mu w^\mu = 0, \tag{38}$$

$$\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu = 0, \tag{39}$$

where w^μ is the four-velocity of an observer at rest relative to the source. The first is equation (5); the second says that the potential is a spacelike covector orthogonal to w^μ; the third holds because $\mathcal{A}_\mu$ is a gradient. The co-expansion factor of an isolated mass is therefore the pure-gauge sector, $\mathcal{F}_{\mu\nu} = 0$, of the five-dimensional metric. The general potential is $\mathcal{A}_\mu = \partial_\mu y + a_\mu$, with field strength $\mathcal{F}_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, and a non-zero a_μ is the only way for the five-dimensional geometry to carry a field strength.

Rotation. For a slowly rotating source, a_μ is the gravitomagnetic field. The field equations are those of general relativity, so at first order in the angular momentum J [31] space is dragged around the source at the angular velocity $\omega = 2GJ/(c^2R^3)$ [37], and the exact Kerr geometry can

be written as a flow of space with a rotational component [19]. The azimuthal flow $\mathbf{v}_s = \omega\,\hat{\mathbf{z}} \times \mathbf{x}$ is the non-gradient component,

$$a_i \simeq \frac{v_{s,i}}{c}, \qquad \mathcal{F} = \nabla \times \mathbf{a} = \frac{2GJ}{c^3}\,\frac{3(\hat{\mathbf{z}}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \hat{\mathbf{z}}}{R^3}, \tag{40}$$

the vorticity of the flow, which has the form of a dipole field and is the gravitomagnetic field of the membrane paradigm [55]. A gyroscope at rest precesses at $\boldsymbol{\Omega}_{\rm LT} = G\left[3(\mathbf{J}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{J}\right]/(c^2R^3)$, which for the orbit of Gravity Probe B gives the general-relativistic 39.2 mas yr^{-1}, against the measured 37.2 ± 7.2 mas yr^{-1} [21].

5.3 The Scalar Sector

With $G_{44} = \Phi^2$, the cylinder condition and $c = 1$, the five-dimensional Einstein equations $G^{(5)}_{AB} = \kappa_5 T^{(5)}_{AB}$ reduce in the standard way [48] to

$$G^{(4)}_{\mu\nu} = \kappa_5 T^{(5)}_{\mu\nu} + \frac{1}{\Phi}\left(\nabla_\mu\nabla_\nu\Phi - g_{\mu\nu}\Box\Phi\right), \qquad \Box\Phi = -\frac{\kappa_5\Phi}{3}\left(\epsilon - 3p + 2p_y\right), \tag{41}$$

for a perfect fluid with energy density ϵ, pressure p and stress p_y along y. This is Brans–Dicke theory [12] with $\omega_{\rm BD} = 0$. Outside a body the scalar obeys $\Box\Phi = 0$. A sourced scalar with $\omega_{\rm BD} = 0$ gives $\gamma_{\rm PPN} = \frac{1}{2}$, excluded by the Cassini value $\gamma_{\rm PPN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [9], so Φ must be constant outside the body to that accuracy. The scalar is switched off only for the stress $p_y = (3p - \epsilon)/2$, which is a fine-tuning condition rather than a result. With Φ constant, equation (41) returns the Schwarzschild exterior, which is the geometry of section 3. The five-dimensional reduction therefore adds nothing to the four-dimensional field equations outside matter, and a dynamics of y of its own, beyond its role as the inclination of space, remains open.

6 Comparison with Experiment

6.1 Gravity

The field equations of the construction are those of general relativity (subsection 3.3), and the geometry of an isolated mass is the Schwarzschild exterior (subsection 3.5). Table 2 lists the main tests.

Table 2. Tests of gravity. The predictions of the construction are those of general relativity.

Test	Prediction	Measurement
Light deflection, Shapiro delay	$\gamma_{\rm PPN} = 1$	$\gamma_{\rm PPN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ (Cassini) [9]
Gravitational redshift	$1/\gamma$	$(0.19 \pm 2.48) \times 10^{-5}$ (Galileo) [17]
Perihelion advance	$6\pi GM/[a(1-e^2)c^2]$	Mercury, $\beta_{\rm PPN} = 1$ [61]
Preferred frame	$\alpha_1 = \alpha_2 = 0$	$\lvert\alpha_1\rvert < 10^{-4}$, $\lvert\alpha_2\rvert < 2 \times 10^{-9}$ [61]
Frame dragging	39.2 mas yr^{-1}	37.2 ± 7.2 mas yr^{-1} (GP-B) [21]
Strong equivalence principle	$\Delta = 0$	$(0.5 \pm 1.8) \times 10^{-6}$ (PSR J0337+1715) [59]
Speed of gravitational waves	c	within -3×10^{-15}, $+7 \times 10^{-16}$ [1]
Ringdown, fundamental mode	$\delta\hat{f}_{220} = \delta\hat{\tau}_{220} = 0$	0.02 ± 0.03, 0.13 ± 0.11 (GWTC-3) [38]
Photon sphere, shadow, ISCO	$3M$, $3\sqrt{3}M$, $6M$	–

6.2 Self-Gravitating Bodies

The weak equivalence principle follows from co-expansion: every kind of matter co-expands by the same factor (subsection 3.1). The strong equivalence principle, that the gravitational binding energy of a body falls like everything else, follows from the field equations: general relativity satisfies it for bodies of any compactness, and the Nordtvedt parameter vanishes. The tests are demanding. Lunar laser ranging limits $|\eta_N|$ to the order of 10^{-4}, where the fractions of self-energy are 4.6×10^{-10} for the Earth and 0.2×10^{-10} for the Moon [61]. For a neutron star the fraction is about 0.1, and the pulsar in the triple system PSR J0337+1715 falls like its white-dwarf companion to $\Delta = (0.5 \pm 1.8) \times 10^{-6}$ [59].

This test is what fixes the direction of co-expansion. If matter co-expanded by γ in every direction, with clocks at $1/\gamma$, the static geometry would agree with general relativity at first post-Newtonian order but not beyond. With the simplest field equation for that geometry, the gravitational mass of a body would fall short of its total energy E by $\int \gamma^2(\epsilon + 3p)(\gamma - 1)^2/(1 + \gamma^2)\, d^3x$, for a fluid of energy density ϵ and pressure p, about 4% for a neutron star of $1.44M_\odot$. That is excluded by PSR J0337+1715 by four orders of magnitude.

Co-expansion along the flow, with lengths across it unchanged, is the choice that agrees with SR and with these tests.

6.3 Special Relativity and Co-scaling

The most direct tests of the absolute frame are the experiments that test SR itself. Subsection 2.3 shows that a clock moving relative to the absolute frame runs at $1/\gamma$, and that the separation it assigns to two events at equal τ is γ times the absolute one. In the absolute-frame reading every physical scale therefore co-scales with the Lorentz factor of the velocity v relative to that frame. Writing the clock-rate, longitudinal and transverse coefficients of the transformation as γ^{n_c}, $\gamma^{n_\parallel}$ and $\gamma^{n_\perp}$ (coefficients of the transformation, not rod lengths: a rod moving through the absolute frame is shortened there by $1/\gamma$), and expanding $\gamma^n = 1 + \frac{n}{2}v^2/c^2$, gives the factors of the Robertson–Mansouri–Sexl test theory [41], $a = 1 + \alpha v^2/c^2$, $b = 1 + \beta v^2/c^2$ and $d = 1 + \delta v^2/c^2$, with

$$(\alpha, \beta, \delta) = \tfrac{1}{2}\left(n_c, n_\parallel, n_\perp\right). \tag{42}$$

Special relativity is $(\alpha, \beta, \delta) = (-\frac{1}{2}, \frac{1}{2}, 0)$. The co-scaling of this construction, with exponents $(-1, +1, 0)$, is therefore special relativity written in its absolute frame, and the experiments that test the parameters of equation (42) test it directly:

- clock rates: a network of optical clocks gives $|\alpha + \frac{1}{2}| \lesssim 1.1 \times 10^{-8}$ [18], so $n_c = -1$ within 2.2×10^{-8};
- clocks against lengths along the motion (Kennedy–Thorndike): $\beta - \alpha - 1 = (-4.8 \pm 3.7) \times 10^{-8}$ [56], so $n_\parallel = +1$ within about 3×10^{-7};
- lengths along against across the motion (Michelson–Morley): $\beta - \delta - \frac{1}{2} = (-0.6 \pm 2.1 \pm 1.2) \times 10^{-10}$ [3, 54], so $n_\perp = 0$ within about 3×10^{-7}.

Co-expansion in gravity has the same exponents, with the flow in place of the motion: clocks at $1/\gamma$, confirmed by the Galileo redshift test, lengths along the flow scaled by γ and lengths across it unchanged, which the Cassini value of γ_{PPN} confirms. Special relativity and gravity are in this sense one co-scaling. A co-scaling of atomic and orbital sizes over cosmic time would appear as an apparent change of G. Lunar laser ranging gives $\dot{G}/G = (7.1 \pm 7.6) \times 10^{-14}\ \mathrm{yr}^{-1}$ [32], about $10^{-3}H_0$, so the two must co-scale identically to that level; a universal co-scaling is not observable by any of these tests.

Because the construction predicts exactly these exponents, the experiments test it in the same way as they test SR: a detected departure would falsify both, not support the absolute frame. A departure would appear as an annual modulation between clocks and lengths of 2.5×10^{-7} times the departure in $\beta - \alpha - 1$, because the velocity of the Earth relative to the cosmic frame, about $370\ \mathrm{km\,s^{-1}}$, changes by $\pm 30\ \mathrm{km\,s^{-1}}$ over a year.

7 Discussion

7.1 What Has Been Shown

Table 3 separates the results of this paper by their standing.

In SR the construction is a reformulation with the same local predictions, and in gravity it is a reformulation of general relativity with the same predictions. In both it changes the account of time. Space has four displacement axes, and motion along them is primitive. Every interval is a sum of squares. The clock rate is the coordinate density $\cos\psi$, both for a moving clock and for a clock at rest near a mass; time readings are ratios of distances along paths; and c is the speed of the causal sphere through which every reading is made.

7.2 The Actual and the Apparent Rate of Time

The model distinguishes two rates. The apparent rate is time as read through the causal sphere, $c\,d\tau$, which every clock comparison uses. The actual rate is the motion along the four axes, which subsection 2.2 shows is not bound to c. In the present formulation the two are tied together, since every time reading is formed from the path through equation (8) and compared through light. The distinction becomes physics only if some process reaches the actual rate independently of light, and the constraints on such a process are strong. Clocks follow the $1/\gamma$ of light's c to about 2×10^{-8} (subsection 6.3). Gravitational waves travel at c to within -3×10^{-15} and $+7 \times 10^{-16}$ of it [1]. Lunar laser ranging ties atomic and orbital rates together to about 10^{-3} of the expansion rate (subsection 6.3). Any difference between the two rates must be invisible to all of these, or lie in a sector they do not test. Ephemeris time [7], time defined from the total change of a system rather than from a clock, is the established precedent for treating the rate of change as primary.

Table 3. Standing of the results

Standing	Results
Identities of the geometry	the inclination of a path and the time readings (subsection 2.2); the Proposition (subsection 2.3); the positive-definite form of every interval; the equality of the height of the inclined space and the synchronization offset (subsection 3.4); that offset as the exact difference between the static and absolute descriptions without a time coordinate (section 4)
Postulates and definitions	that **S** is physically distinguished; that space flows outward; co-expansion of matter along the flow
Derived	the flow rule as the Hamiltonian constraint on flat absolute slices, with the Hubble flow as the same equation; the inclined space as Flamm's paraboloid; the static clock rate $1/\gamma$ and the measured geometry of an isolated mass, which is the Schwarzschild exterior; the weak equivalence principle from co-expansion; orbits, the photon sphere and light deflection without a time coordinate; the co-scaling exponents of gravity, equal to those of SR
Taken from general relativity	the field equations, and with them the strong equivalence principle, rotation, radiation and cosmology
Not established	a slicing of absolute time for several bodies and for collapse; a dynamics of y of its own; the co-expansion form of an expanding universe

7.3 Relation to Other Formulations

The classical simultaneity of subsection 2.3 places this construction in a known family. Mansouri and Sexl [41] constructed an ether theory that keeps absolute simultaneity and is kinematically equivalent to special relativity. Equation (11) is of that type, with the offset between absolute and Einstein synchronization, parametrized by Reichenbach's ϵ [52], carried by the coordinate y. Malament [40] showed that, given only the causal structure and an inertial observer, Einstein's $\epsilon = \frac{1}{2}$ is the only definable simultaneity relation. A different, absolute simultaneity therefore needs structure beyond the causal one. Here that structure is the flow frame of section 3.2, which is assumed rather than derived.

Around a mass the absolute frame is the outward flow of equation (16), the time reverse of the river model [30], and its inclination into y is Flamm's embedding of the Schwarzschild space [24]. Theories in which a preferred foliation is dynamical, Einstein–aether theory [33] and the khronometric models [10], predict preferred-frame effects, bounded by $|\alpha_1| < 10^{-4}$ from lunar laser ranging, $|\alpha_1| < 7 \times 10^{-5}$ from PSR J1738+0333, and $|\alpha_2| < 2 \times 10^{-9}$ from millisecond pulsars [61]. Here the absolute slices are a slicing of the geometry of general relativity and no field equation refers to them, so $\alpha_1 = \alpha_2 = 0$. Rotating sources have the same structure: the Kerr geometry can be written as a flow of space, with a rotational component, on slices of the same kind [19].

The positive-definite geometry of subsection 2.2 is related to Euclidean relativity [46, 2], and the description of steady gravitational fields without a time coordinate to the optical and Jacobi geometries of stationary spacetimes [27, 26]. Shape dynamics [28] shows that general relativity admits a reformulation with a preferred foliation and no spacetime, and it is the closest established framework for the slicing of several bodies.

7.4 Limitations

The construction takes its field equations from general relativity rather than deriving them from a dynamics of y. The slicing of absolute time is specified for an isolated mass and for a homogeneous universe, but not for several bodies, where how the absolute slices of different bodies join is open, nor for gravitational collapse, where the outward flow covers the time reverse of a black-hole interior. Inside a body supported by pressure the rate of absolute time differs from one. Time is removed only for steady flows, and an expanding universe with varying H and gravitational waves are not described without a time coordinate here. A body at rest has no path, so its own time must come from internal motion or from the change of everything else, and light is a degenerate limit.

7.5 Conclusion

Relativistic motion and gravitation can be described without a time dimension, with motion along four displacement axes as the fundamental rate of change and every interval a sum of squares. One inclination, of a path in SR and of space around a mass in gravity, gives the Lorentz factor, the speed, the coordinate density and the clock rate. Time readings are ratios of distances, c is the speed of the causal sphere through which they are made, and every inertial observer recovers one absolute time. Matter at rest relative to a mass co-expands along the flow of space, as a moving body does in SR, and the universality of that co-expansion is the equivalence principle. On flat

absolute slices the flow is set by the enclosed mass through the Hamiltonian constraint of the Einstein equations, the inclination of space into y is Flamm's paraboloid, its height is the synchronization offset between absolute and static clocks, and light and matter move without a time coordinate. The geometry is that of general relativity, so the construction passes every test that general relativity passes, and gives it an absolute time.

Appendix A Orbits without a Time Coordinate

This appendix derives the orbit results of section 4 without a time coordinate. Units with $c = G = 1$ are used. Write equation (22) with $A = 1/\gamma^2 = 1 - 2M/R$ and $B = \gamma^2 = 1/A$, so that $AB = 1$. For a static geometry a body of unit mass and energy E follows the geodesics of the Jacobi metric [26], which in the equatorial plane is

$$J = \frac{E^2 - A}{A}\left(B\,dR^2 + R^2 d\phi^2\right), \tag{43}$$

equation (30). The metric does not depend on ϕ, so along a geodesic the momentum conjugate to ϕ is conserved; it equals the angular momentum L per unit mass. Eliminating the arc length,

$$\left(\frac{dR}{d\phi}\right)^2 = \frac{R^4}{AB\,L^2}\left[E^2 - A\left(1 + \frac{L^2}{R^2}\right)\right], \tag{44}$$

which with $u = 1/R$ and $AB = 1$ is equation (31), the Schwarzschild orbit equation [45].

Circular orbits and stability. A circular orbit is a double root of $f_0(u) = E^2 - (1 - 2Mu)(1 + L^2u^2)$. The conditions $f_0 = f_0' = 0$ give $L^2 = MR^2/(R - 3M)$, existing for $R > 3M$, and the orbit is marginally stable where the root is triple, at $R = 6M$.

Perihelion advance. For a bound orbit with semi-major axis a and eccentricity e the angle between successive perihelia is

$$\Delta\phi = \frac{6\pi M}{a(1 - e^2)} + O\left(\frac{M^2}{a^2}\right), \tag{45}$$

which is $6\pi GM/[a(1 - e^2)c^2]$ in physical units.

Light. For light the Jacobi metric reduces to the optical metric $\gamma^2(\gamma^2 dR^2 + R^2 d\phi^2)$, and the same steps give equation (32), whose deflection is equation (33).

Absolute frame. In the absolute frame the same paths follow from the Randers form of the travel time, which differs from the static form by the exact differential of equation (29) for light, and by E times it for bodies.

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Author contributions
The author conceived, developed, conducted, and wrote this research independently.

Data availability
No data were generated or analysed in this study. The Python code that reproduces and checks the results of this paper, together with a notebook that walks through them, is available in the alpha-omega-gravity package, https://pypi.org/project/alpha-omega-gravity/; the results on co-expansion along the flow are checked in its module `paper_two.flow_geometry`.

Declaration of generative AI use
While the theory described here was developed independently, during the preparation of this manuscript the author used Claude (Anthropic) to review the manuscript, to check and extend derivations, to draft and edit text, and to write the Python code that verifies the results and generates the accompanying notebook. The author reviewed and edited all AI-assisted content and takes full responsibility for the content of the publication.

References

[1] B. P. Abbott et al. "Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A". In: *The Astrophysical Journal Letters* 848.2 (2017), p. L13. DOI: 10.3847/2041-8213/aa920c.

[2] José B. Almeida. *4-Dimensional Optics, an Alternative to Relativity*. 2001. arXiv: gr-qc/0107083 [gr-qc].

[3] Paul Antonini et al. "Test of Constancy of Speed of Light with Rotating Cryogenic Optical Resonators". In: *Physical Review A* 71 (2005), p. 050101. DOI: 10.1103/PhysRevA.71.050101.

[4] Richard Arnowitt, Stanley Deser, and Charles W. Misner. "Republication of: The dynamics of general relativity". In: *General Relativity and Gravitation* 40 (2008). Originally published in L. Witten (ed.), Gravitation: An Introduction to Current Research, Wiley, 1962, pp. 227–265, pp. 1997–2027. DOI: 10.1007/s10714-008-0661-1.

[5] J. Bailey et al. "Final Report on the CERN Muon Storage Ring Including the Anomalous Magnetic Moment and the Electric Dipole Moment of the Muon, and a Direct Test of Relativistic Time Dilation". In: *Nuclear Physics B* 150 (1979), pp. 1–75. DOI: 10.1016/0550-3213(79)90292-X.

[6] J. Bailey et al. "Measurements of Relativistic Time Dilatation for Positive and Negative Muons in a Circular Orbit". In: *Nature* 268 (1977), pp. 301–305. DOI: 10.1038/268301a0.

[7] Julian Barbour. *The Nature of Time*. First juried prize, FQXi essay competition on the nature of time. 2009. arXiv: 0903.3489 [gr-qc].

[8] John S. Bell. "How to Teach Special Relativity". In: *Speakable and Unspeakable in Quantum Mechanics*. First published in Progress in Scientific Culture 1 (1976). Cambridge: Cambridge University Press, 1987, pp. 67–80.

[9] Bruno Bertotti, Luciano Iess, and Paolo Tortora. "A test of general relativity using radio links with the Cassini spacecraft". In: *Nature* 425.6956 (2003), pp. 374–376. DOI: 10.1038/nature01997.

[10] Diego Blas, Oriol Pujolàs, and Sergey Sibiryakov. "Models of non-relativistic quantum gravity: the good, the bad and the healthy". In: *Journal of High Energy Physics* 2011.4 (2011), p. 018. DOI: 10.1007/JHEP04(2011)018.

[11] Benjamin Botermann et al. "Test of Time Dilation Using Stored Li+ Ions as Clocks at Relativistic Speed". In: *Physical Review Letters* 113.12 (2014), p. 120405. DOI: 10.1103/PhysRevLett.113.120405.

[12] C. Brans and R. H. Dicke. "Mach's Principle and a Relativistic Theory of Gravitation". In: *Physical Review* 124 (1961), pp. 925–935.

[13] Dieter R. Brill and Richard W. Lindquist. "Interaction Energy in Geometrostatics". In: *Physical Review* 131 (1963), pp. 471–476. DOI: 10.1103/PhysRev.131.471.

[14] Harvey R. Brown. *Physical Relativity: Space-Time Structure from a Dynamical Perspective*. Oxford: Oxford University Press, 2005.

[15] John Edward Campbell. *A Course of Differential Geometry*. Oxford: Clarendon Press, 1926.

[16] Sumanto Chanda et al. *Jacobi–Maupertuis Randers–Finsler Metric for Curved Spaces and the Gravitational Magnetoelectric Effect*. 2019. arXiv: 1903.11805 [gr-qc].

[17] Pacome Delva et al. "Gravitational Redshift Test Using Eccentric Galileo Satellites". In: *Physical Review Letters* 121 (2018), p. 231101. DOI: 10.1103/PhysRevLett.121.231101.

[18] Pacome Delva et al. "Test of Special Relativity Using a Fiber Network of Optical Clocks". In: *Physical Review Letters* 118 (2017), p. 221102. DOI: 10.1103/PhysRevLett.118.221102.

[19] Chris Doran. "A new form of the Kerr solution". In: *Physical Review D* 61 (2000), p. 067503. DOI: 10.1103/PhysRevD.61.067503.

[20] Albert Einstein. "On the Electrodynamics of Moving Bodies". In: *Annalen der Physik* 17.10 (1905), pp. 891–921. DOI: 10.1002/andp.19053221004.

[21] C. W. F. Everitt et al. "Gravity Probe B: Final Results of a Space Experiment to Test General Relativity". In: *Physical Review Letters* 106 (2011), p. 221101.

[22] Valerio Faraoni. "Cosmological apparent and trapping horizons". In: (2011). eprint: 1106.4427.

[23] Valerio Faraoni et al. "Friedmann–Lemaître–Robertson–Walker cosmology through the lens of gravitoelectromagnetism". In: (2022). eprint: 2201.05287.

[24] Ludwig Flamm. "Beiträge zur Einsteinschen Gravitationstheorie". In: *Physikalische Zeitschrift* 17 (1916), pp. 448–454.

[25] David H. Frisch and James H. Smith. "Measurement of the Relativistic Time Dilation Using Mu-Mesons". In: *American Journal of Physics* 31.5 (1963), pp. 342–355. DOI: 10.1119/1.1969508.

[26] G. W. Gibbons. "The Jacobi-Metric for Timelike Geodesics in Static Spacetimes". In: *Classical and Quantum Gravity* 33 (2016), p. 025004. arXiv: 1508.06755.

[27] G. W. Gibbons et al. "Stationary Metrics and Optical Zermelo–Randers–Finsler Geometry". In: *Physical Review D* 79 (2009), p. 044022. DOI: 10.1103/PhysRevD.79.044022.

[28] Henrique Gomes, Sean Gryb, and Tim Koslowski. "Einstein Gravity as a 3D Conformally Invariant Theory". In: *Classical and Quantum Gravity* 28 (2011), p. 045005. arXiv: 1010.2481.

[29] Allvar Gullstrand. "Allgemeine Lösung des statischen Einkörperproblems in der Einsteinschen Gravitationstheorie". In: *Arkiv för Matematik, Astronomi och Fysik* 16.8 (1922), pp. 1–15.

[30] Andrew J. S. Hamilton and Jason P. Lisle. "The river model of black holes". In: *American Journal of Physics* 76 (2008), pp. 519–532. DOI: 10.1119/1.2830526.

[31] James B. Hartle. "Slowly Rotating Relativistic Stars. I. Equations of Structure". In: *The Astrophysical Journal* 150 (1967), p. 1005. DOI: 10.1086/149400.

[32] Franz Hofmann and Jürgen Müller. "Relativistic Tests with Lunar Laser Ranging". In: *Classical and Quantum Gravity* 35.3 (2018), p. 035015. DOI: 10.1088/1361-6382/aa8f7a.

[33] Ted Jacobson and David Mattingly. "Gravity with a dynamical preferred frame". In: *Physical Review D* 64 (2001), p. 024028. DOI: 10.1103/PhysRevD.64.024028.

[34] Theodor Kaluza. "Zum Unitätsproblem der Physik". In: *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin (Math. Phys.)* (1921), pp. 966–972.

[35] Oskar Klein. "Quantentheorie und fünfdimensionale Relativitätstheorie". In: *Zeitschrift für Physik* 37.12 (1926), pp. 895–906. DOI: 10.1007/BF01397481.

[36] Georges Lemaître. "L'Univers en expansion". In: *Annales de la Société Scientifique de Bruxelles A* 53 (1933), pp. 51–85.

[37] J. Lense and H. Thirring. "Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie". In: *Physikalische Zeitschrift* 19 (1918), pp. 156–163.

[38] LIGO Scientific Collaboration, Virgo Collaboration, and KAGRA Collaboration. "Tests of General Relativity with GWTC-3". In: *Physical Review D* 112 (2025), p. 084080. DOI: 10.1103/PhysRevD.112.084080. arXiv: 2112.06861.

[39] L. Magaard. "Zur Einbettung Riemannscher Räume in Einstein-Räume und konforme Einbettung Riemannscher Räume". PhD thesis. Universität Kiel, 1963.

[40] David Malament. "Causal theories of time and the conventionality of simultaneity". In: *Noûs* 11.3 (1977), pp. 293–300.

[41] Reza Mansouri and Roman U. Sexl. "A test theory of special relativity: I. Simultaneity and clock synchronization". In: *General Relativity and Gravitation* 8.7 (1977), pp. 497–513. DOI: 10.1007/BF00762634.

[42] Karl Martel and Eric Poisson. "Regular coordinate systems for Schwarzschild and other spherical spacetimes". In: *American Journal of Physics* 69 (2001), p. 476. eprint: gr-qc/0001069.

[43] W. H. McCrea and E. A. Milne. "Newtonian universes and the curvature of space". In: *Quarterly Journal of Mathematics (Oxford)* 5 (1934), pp. 73–80.

[44] E. A. Milne. "A Newtonian expanding universe". In: *Quarterly Journal of Mathematics (Oxford)* 5 (1934), pp. 64–72.

[45] Charles W. Misner, Kip S. Thorne, and John A. Wheeler. *Gravitation*. San Francisco: W. H. Freeman, 1973.

[46] Hans Montanus. "Proper-Time Formulation of Relativistic Dynamics". In: *Foundations of Physics* 31.9 (2001), pp. 1357–1400. DOI: 10.1023/A:1012274211780.

[47] J. R. Oppenheimer and G. M. Volkoff. "On Massive Neutron Cores". In: *Physical Review* 55 (1939), pp. 374–381. DOI: 10.1103/PhysRev.55.374.

[48] J. M. Overduin and P. S. Wesson. "Kaluza-Klein Gravity". In: *Physics Reports* 283 (1997), pp. 303–378.

[49] Paul Painlevé. "La mécanique classique et la théorie de la relativité". In: *Comptes Rendus de l'Académie des Sciences (Paris)* 173 (1921), pp. 677–680.

[50] Lisa Randall and Raman Sundrum. "An Alternative to Compactification". In: *Physical Review Letters* 83 (1999), pp. 4690–4693. DOI: 10.1103/PhysRevLett.83.4690.

[51] Amalkumar Raychaudhuri. "Relativistic Cosmology. I". In: *Physical Review* 98.4 (1955), pp. 1123–1126. DOI: 10.1103/PhysRev.98.1123.

[52] Hans Reichenbach. *The Philosophy of Space and Time*. New York: Dover, 1958.

[53] Bruno Rossi and David B. Hall. "Variation of the Rate of Decay of Mesotrons with Momentum". In: *Physical Review* 59.3 (1941), pp. 223–228. DOI: 10.1103/PhysRev.59.223.

[54] Stephan Schiller, Paul Antonini, and Michael Okhapkin. "A Precision Test of the Isotropy of the Speed of Light Using Rotating Cryogenic Optical Cavities". In: *Special Relativity: Will it Survive the Next 101 Years?* Ed. by Jürgen Ehlers and Claus Lämmerzahl. Vol. 702. Lecture Notes in Physics. Berlin: Springer, 2006.

[55] Kip S. Thorne, Richard H. Price, and Douglas A. Macdonald, eds. *Black Holes: The Membrane Paradigm*. New Haven: Yale University Press, 1986.

[56] Michael E. Tobar et al. "Testing Local Lorentz and Position Invariance and Variation of Fundamental Constants by Searching the Derivative of the Comparison Frequency between a Cryogenic Sapphire Oscillator and Hydrogen Maser". In: *Physical Review D* 81 (2010), p. 022003. DOI: 10.1103/PhysRevD.81.022003.

[57] Richard C. Tolman. "Effect of inhomogeneity on cosmological models". In: *Proceedings of the National Academy of Sciences* 20 (1934), pp. 169–176.

[58] Richard C. Tolman. "Static Solutions of Einstein's Field Equations for Spheres of Fluid". In: *Physical Review* 55 (1939), pp. 364–373. DOI: 10.1103/PhysRev.55.364.

[59] G. Voisin et al. "An improved test of the strong equivalence principle with the pulsar in a triple star system". In: *Astronomy & Astrophysics* 638 (2020), A24. DOI: 10.1051/0004-6361/202038104.

[60] Paul S. Wesson. *Space-Time-Matter: Modern Kaluza-Klein Theory*. Singapore: World Scientific, 1999.

[61] Clifford M. Will. "The confrontation between general relativity and experiment". In: *Living Reviews in Relativity* 17 (2014), p. 4. DOI: 10.12942/lrr-2014-4.